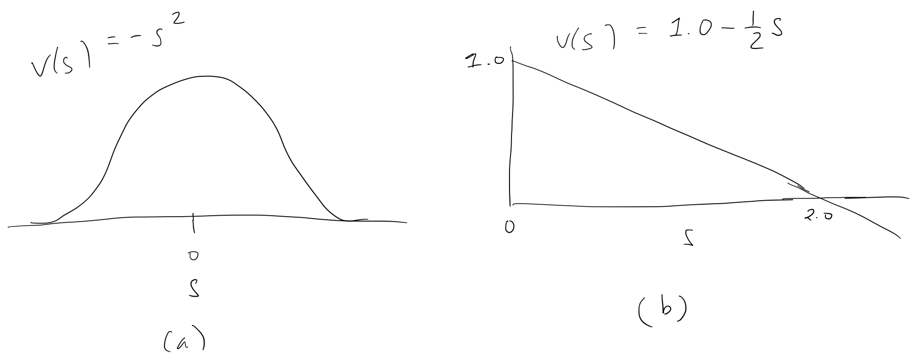
Worksheet 11

1. Consider the following two functions.



(a) Design features for each function, to approximate them as a linear function of these features. Can you design features to make the approximation exact?

(b) Can you design one set of features, that allows you to represent both functions?

2. Consider the following neural network $\hat{v}$ with one-hidden layer, relu activation function g, with weights $\mathbf{W}^{[0]}, \mathbf{W}^{[1]}, \mathbf{b}^{[0]}, \mathbf{b}^{[1]}$,

$$\hat{v}(\mathbf{x}; \mathbf{W}^{[0]}, \mathbf{W}^{[1]}, \mathbf{b}^{[0]}, \mathbf{b}^{[1]}) = \mathbf{W}^{[1]} g(\mathbf{W}^{[0]}\mathbf{x} + \mathbf{b}^{[\mathbf{0}]}) + \mathbf{b}^{[1]}.$$

Recall that the following gradients

$$\frac{\partial \hat{v}}{\partial \mathbf{W}^{[1]}_{ij}} = g(\mathbf{W}^{[0]}\mathbf{x} + \mathbf{b}^{[\mathbf{0}]})_j$$

$$\frac{\partial \hat{v}}{\partial \mathbf{b}^{[1]}_{j}} = 1$$

$$\frac{\partial \hat{v}}{\partial \mathbf{b}^{[0]}_{i}} = \sum_k \mathbf{W}^{[1]}_{ki} \frac{\partial g(\mathbf{W}^{[0]}\mathbf{x} + \mathbf{b}^{[0]})}{\partial \mathbf{b}^{[0]}_{i}}$$

$$\frac{\partial \hat{v}}{\partial \mathbf{W}^{[0]}_{ij}} = \sum_k \mathbf{W}^{[1]}_{ki} \frac{\partial g(\mathbf{W}^{[0]}\mathbf{x} + \mathbf{b}^{[\mathbf{0}]})}{\partial \mathbf{W}^{[0]}_{ij}}$$

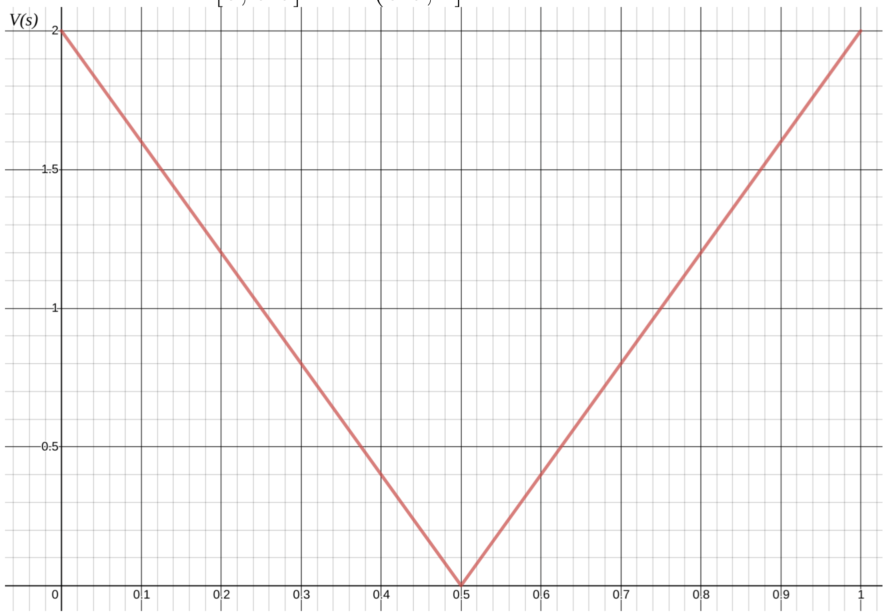
(a) What are the derivatives specifically for the relu activation g?

(b) We talked about carefully initializing the weights for the NN. For example, each weight can be sampled from a Gaussian distribution. Imagine instead you decided to initialize all the weights to zero. Why would this be a problem? Hint: Consider the derivatives in (a).

3. Consider a problem with the state space, $\mathcal{S} = \{0, 0.01, 0.02, \cdots, 1\}$. Assume the true value function is

$$v_\pi(s) = 4|s - 0.5|$$

which is visualized below. We decide to create features with state aggregation, and choose to aggregate into two bins: $[0, 0.5]$ and $(0.5, 1]$.



(a) What are the possible feature vectors for this state aggregation?

(b) Imagine you minimize the $\overline{\text{VE}}(\mathbf{w}) = \sum_{s \in \mathcal{S}} d(s)(v_\pi(s) - \hat{v}(s, \mathbf{w}))^2$ with a uniform weighting $d(s) = \frac{1}{101}$ for all $s \in \mathcal{S}$. What vector $\mathbf{w}$ is found?

(c) Now, if the agent puts all of the weighting on the range $[0, 0.25]$, (i.e. $d(s) = 0$ for all $s \in (0.25, 1]$), then what vector $\mathbf{w}$ is found by minimizing $\overline{\text{VE}}$?

4. Consider the following general SGD update rule with a general target U_t

$$\mathbf{w}_{t+1} = \mathbf{w}_t + \alpha \left[U_t - \hat{v}(S_t, \mathbf{w}_t)\right] \nabla \hat{v}(S_t, \mathbf{w_t}).$$

Assume we are using linear function approximation, *i.e.* $\hat{v}(S, \mathbf{w}) = x(S)^\top \mathbf{w}$.

(a) What happens to the update if we scale the features by a constant and use the new features $\tilde{x}(S) = 2x(S)$? Why might this be a problem?

(b) In general we want a stepsize that is invariant to the magnitude of the feature vector $x(S)$, where the magnitude is measured by the inner product $x(S)^\top x(S)$. The book suggests the following stepsize:

$$\alpha = \frac{1}{\tau x(S)^\top x(S)}.$$

What is $x(S)^\top x(S)$ when using tile coding with 10 tilings? Suppose $\tau = 1000$, what is α if we use tile coding with 10 tilings?