Worksheet 10

1. Let $f(x, y) = (x + y)^2 + e^{xy}$. Recall that the gradient is composed of the partial derivatives for each variable

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

where $\frac{\partial f(x,y)}{\partial x}$ is the derivative of $f(x, y)$ w.r.t. x assuming that y is fixed.

(a) What is $\nabla f(x, y)$ for the f defined above? Hint: Recall that the derivative of e^z is e^z.

(b) What is $\nabla f(0, 1)$?

2. Find the gradient of f

(a) if $f(x, y, z) = \frac{y^z}{x}$

(b) if $f(x) = e^{x^2+5}$

(c) if $f(\mathbf{x}) = \mathbf{x}^T\beta$ where $\mathbf{x}$ is a vector in $\mathbb{R}^N$ and β is a vector of constants in $\mathbb{R}^N$

(d) if $f(\mathbf{x}) = (\mathbf{x}^T\beta - y)^2$ where $\mathbf{x}$ is a vector in $\mathbb{R}^N$, β is a vector of constants in $\mathbb{R}^N$, and y is a scalar in $\mathbb{R}$

3. (*Exercise 9.1 S&B*) Show that tabular methods such as presented in Course 2 of the MOOC (and Part I of the book) are a special case of linear function approximation. What would the feature vectors be?

4. **Challenge Question:** At the TD fixed point, we have seen that the VE is within a bounded expansion of the lowest possible error:

$$\overline{\mathrm{VE}}(\mathbf{w}_{\mathrm{TD}}) \leq \frac{1}{1-\gamma} \min_{\mathbf{w}} \overline{\mathrm{VE}}(\mathbf{w})$$

(a) If $\gamma = 0.9$ and $\min_{\mathbf{w}} \overline{\mathrm{VE}}(\mathbf{w}) = 1$, what is the minimum and maximum value of $\overline{\mathrm{VE}}(\mathbf{w}_{\mathrm{TD}})$? What is the minimum and maximum value of $\overline{VE}(\mathbf{w}_{\mathrm{TD}})$ if $\min_{\mathbf{w}} \overline{\mathrm{VE}}(\mathbf{w}) = 0$? Hint: recall that $\min_{\mathbf{w}} \overline{\mathrm{VE}}(\mathbf{w})$ is the minimal value error you can achieve under this parameterization.

(b) We have seen that if we can perfectly represent the value function, then $\min_{\mathbf{w}} \overline{VE}(\mathbf{w}) = 0$. How about the other direction: if $\min_{\mathbf{w}} \overline{VE}(\mathbf{w}) = 0$, then does that mean we can represent true value function?